

## Statistical models for aboveground biomass of *Populus deltoides* planted in agroforestry in Haryana

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**Abstract:** Non-linear models were constructed and validated for aboveground biomass of *Populus deltoides* planted in agroforestry in the northern state of Haryana, India. Independent data sets were used for construction and validation of models. Six models were separately fitted using 'D' and 'D<sup>2</sup>H' as explanatory variables; where, H - height of tree (m) and D - diameter at breast height (cm). Both variables were highly correlated with the aboveground biomass (B). The model  $B = 0.936 D^{1.809}$  was found to be a good fit ( $R^2=0.914$ ,  $MSE=840.432$ ). Mean absolute error of predictions was 10.52 kg with model efficiency of 94.9 percent. Besides, the logistic model  $B = 639.186 [1 + \exp(1.515 - 1.261 D^2H)]^{-1}$  was adjudged best ( $R^2=0.963$ ,  $MSE= 375.594$ ) with mean absolute error of predictions 7.20 kg and model efficiency of 98.4 percent. The model fitted using variable 'D<sup>2</sup>H' was better in predictions than model fitted using variable 'D', but the former model may not be used when it is difficult to measure the tree height.

**Resumen:** Se construyeron y se validaron modelos no lineales para la biomasa aérea de *Populus deltoides* plantado en sistemas agroforestales en el norteño estado de Haryana, India. Se usaron conjuntos de datos independientes para la construcción y validación de los modelos. Se ajustaron seis modelos por separado usando 'D' y 'D<sup>2</sup>H' como variables explicativas; donde H = altura del árbol (m) y D = diámetro a la altura del pecho (cm). Ambas variables estuvieron fuertemente correlacionadas con la biomasa aérea (B). Se obtuvo un buen ajuste para el modelo  $B = 0.936 D^{1.809}$  ( $R^2 = 0.914$ ,  $EEM = 840.432$ ). El error absoluto medio de las predicciones fue de 10.52 kg, con una eficiencia del modelo de 94.9 por ciento. Además, el modelo logístico  $B = 639.186 [1 + \exp(1.515 - 1.261 D^2H)]^{-1}$  fue juzgado como el mejor ( $R^2 = 0.963$ ,  $EEM = 375.594$ ), con un error absoluto medio de la predicción de 7.20 kg y una eficiencia del modelo de 98.4 por ciento. El modelo ajustado usando la variable 'D<sup>2</sup>H' fue mejor en términos de las predicciones que el modelo ajustado usando la variable 'D', pero el primer modelo podría no ser utilizable en caso de haber dificultades para medir la altura de los árboles.

**Resumo:** Modelos não lineares foram construídos e validados para a biomassa aérea da *Populus deltoides* plantada num sistema agroflorestal no norte do Estado de Haryana, Índia. Séries de dados independentes foram usadas para a construção e validação dos modelos. Seis modelos foram separadamente ajustados usando 'D' e 'D<sup>2</sup>H' como variáveis explicativas; sendo H - altura da árvore (m) e D - diâmetro à altura do peito (cm). Ambas as variáveis mostraram-se fortemente correlacionadas com a biomassa aérea (B). O modelo  $B = 0.936 D^{1.809}$  mostrou ser um bom ajustamento ( $R^2 = 0.914$ ,  $MSE = 840.432$ ). O erro médio absoluto das predições foi de 7.20 kg sendo a eficiência do modelo igual a 94.9 por cento. O modelo ajustado com a variável

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'D<sup>2</sup>H' foi revelou-se um preditor melhor do que o modelo ajustado com a variável 'D', mas o primeiro modelo pode não ser utilizável quando for difícil a medida da altura da árvore.

**Key words:** Agroforestry, aboveground, biomass, model, *Populus deltoides*, statistical.

## Introduction

Poplar (*Populus deltoides*) are now being raised in India on an increasing scale as an agroforestry crop by the farmers on their field bunds, agricultural fields, vacant lands, etc. for their domestic needs and for sale. Poplars are eminently suited for agroforestry and are being raised in Uttar Pradesh, Uttaranchal, Punjab, Haryana and Jammu & Kashmir. Poplars have been grown on farmlands in single rows along field boundaries, along paths and as blocks. Large scale planting of Poplars by farmers was started since 1979 when WIMCO actively participated in the extension programme of poplar cultivation. WIMCO found it excellent for matchwood.

Poplars are the ideal tree both for the farmers and foresters. Their fast growing and deciduous nature, multipurpose use and compatibility with agricultural crops have made them all the more important for agroforestry plantations in Uttar Pradesh, Haryana and Punjab since 1980 (Chaturvedi 1982). Mathur & Sharma (1983) concluded that raising of Poplar at 8 years rotation with agriculture on farmland is profitable. The benefit-cost ratio worked out to be 3.22 with 12 per cent interest. Chandra (1986) also gave an excellent account of the economics involved in establishing Poplar based agroforestry. The discounted benefits at 12% over 8 years were worked out to Rs.77336/- against the discounted costs of Rs. 41503/-.

Various poplar based agroforestry models adopted by farmers in Haryana (Kumar *et al.* 2004) include:

- (i) Polar-sugarcane-turmeric block plantation
- (ii) Polar-sugarcane-wheat-chari block plantation
- (iii) Polar-sugarcane-wheat-chari-maize-bajra block plantation
- (iv) Polar-sugarcane-potato-berseem-chari block plantation
- (v) Poplar-paddy-wheat boundary plantation

- (vi) Poplar-sugarcane-paddy-wheat boundary plantation

Economic analysis for six years rotation indicates that these agroforestry models were highly lucrative.

Nandal & Hooda (2005) studied the effect of spacing of Poplar on production of interspaced crops viz. sorghum, cowpea, dhaincha, groundnut, moongbean and turmeric. They found that yield of all crops increased with increasing Poplar spacing, however, a spacing of 10 m x 2.5 m was ideal for getting optimum growth and yield of agricultural crops.

Sharma (1979) and Tokey *et al.* (1994) developed linear equations for aboveground biomass (green and dry) of Poplar using D<sup>2</sup>H and D as an independent variables, respectively. But tree growth follows sigmoid/ non-linear pattern, so these equations may not hold good. Moreover, some researchers (Negi & Tandon 1997; Puri 2002) developed allometric/ polynomial equations for aboveground biomass, but their statistical validation was not done properly. Keeping this in view, an attempt has been made in this study to develop non-linear models for predicting aboveground biomass (green) of *Populus deltoides*. Selection and validation of best model was done using an independent data set.

## Materials and methods

Poplar agroforestry plantations have been selected from Karnal, Hissar and Yamunanagar districts of Haryana. The climate of these districts is characterized by extreme dryness of air with an intensely hot summer and cold winter. Soil is Indo-Gangetic alluvial plain which is sandy clay loam in texture. Sample trees were taken from both block and agrisilviculture plantations where the spacing was 5 m x 4 m and 10 m x 2.5 m, respectively. Data sample includes a total of 60 Poplar trees of six (40 trees), seven (10 trees) and eight (10 trees) years. Data were recorded on tree height (H), bole

**Table 1.** Non-linear functions used for aboveground biomass of *Populus deltoides*.

Function No.	Function Name	Function Form	References
[1]	Logistic	$Y = a (1 + \exp(b-cX))^{-1}$	Hutchinson (1978)
[2]	Gompertz	$Y = a \exp(-b.e^{-cX})$	Causton & Venus (1981); Zullinger <i>et al.</i> (1984)
[3]	Chapman-Richards	$Y = a (1 - \exp(-bX))^c$	Richards (1959)
[4]	Weibull	$Y = a \exp(1 - b.\exp(-cX^d))$	Yang <i>et al.</i> (1978)
[5]	Sloboda	$Y = a \exp(-b.\exp(-cX^d))$	Kiviste (1988); Zeide (1993)
[6]	Exponential	$Y = a.X^b$	Harding & Grigal (1985)

height, diameter at breast height (DBH) and total aboveground biomass (branch, bole and leaf) from the harvested trees. Growth and biomass data were recorded using standard meter tape and weighing balance. Data from 6 year old trees were used for development of models, whereas data from trees of 7 and 8 year age were used for validation of models. Snee (1977) recommended that the data splitting is an effective method of model validation when it is not practical to collect the new data to test the model. Six non-linear functions were fitted using both D and D<sup>2</sup>H as explanatory variables separately for aboveground biomass (Table 1). Non-linear regression procedure of SYSTAT 9.0 was applied and the parameters were estimated using least square method (Wilkinson *et al.* 1996). Normality of explanatory variables was checked by W-test given by Shapiro and Wilk (Rao *et al.* 1985). Model validation was done through graphical and statistical methods as suggested by Janssen & Heuberger (1995), Mayer & Butler (1993), Reynolds & Chung (1986) and Rawlings (1988). Statistical criteria such as mean square error (MSE), adjusted R<sup>2</sup>, asymptotic standard error (ASE) and Wald Confidence Intervals (95%) were used for judging the adequacy of the fitted model. Modified error index as suggested by Reynolds *et al.* (1988) was also applied for the selection and/or assessment of final selected model.

## Results and discussion

The descriptive statistics of the sample trees are presented in Table 2. Height of sample trees varied from 13.40 to 24.95 m with an average of 18.95 m. The coefficient of variation indicated 16.5% variation in tree height. The Shapiro-Wilk p-value of 0.189 confirms the normality of this variable. Another important explanatory variable is the diameter at breast height (DBH). Values of DBH for sampled trees ranged from 16.56 to 35.03

cm with an average value of 25.88 cm. The variation in this variable was found to be 20.8 per cent. Shapiro-Wilk p-value (0.018) indicated that distribution is normal.

Bole, branch and leaf weights were added to get the aboveground biomass (B). For the sampled trees, this biomass varied from 158.32 to 622.64 kg with an average value of 351.17 kg. Variation was found to be high (CV=37.9%) for the aboveground biomass. SW p-value indicated that distribution is non-normal. Correlation of the height with aboveground biomass was significantly high ( $r = 0.986$ ). The diameter at breast height was also highly correlated with aboveground biomass ( $r = 0.971$ ).

### Aboveground Biomass – DBH Relationship

Models were constructed for aboveground

**Table 2.** Descriptive statistics for the sample trees of *Populus deltoids* (n=60).

Statistics	Tree height (m)	DBH (cm)	AGBM (kg)
Minimum	13.40	16.56	158.32
Maximum	24.95	35.03	622.64
Mean	18.96	25.88	351.17
C.V. (%)	16.50	20.80	37.90
SW p-value	0.189	0.018	0.007

DBH- Diameter at breast height; AGBM- Aboveground biomass

**Table 3.** Mean square error (MSE) and adjusted R<sup>2</sup> of the fitted functions.

Function No.	Using 'D'		Using 'D <sup>2</sup> H'	
	MSE	Adj. R <sup>2</sup>	MSE	Adj. R <sup>2</sup>
[1]	794.368	0.919	375.594	0.963
[2]	816.051	0.916	359.682	0.964
[3]	900.436	0.908	393.041	0.960
[4]	838.163	0.916	360.615	0.965
[5]	835.636	0.917	360.615	0.965
[6]	840.432	0.914	379.748	0.961

biomass of Poplar tree using diameter at breast height as explanatory variable. Six non-linear functions were fitted (Table 1) for the aboveground biomass. The mean square error (MSE) and adjusted  $R^2$  obtained for fitted functions are given in Table 3. It can be observed from the table that MSE is maximum for function [3] i.e. 900.436 and minimum for function [1] i.e. 794.368. The value of adj.  $R^2$  was highest (0.919) for function [1] followed by function [5] i.e. 0.917.

The estimated parameters and their asymptotic standard error (ASE) are presented in Table 4. The estimates of parameter 'a' of functions [1] and [3] seem suspect as their ASE was very high. The ASE for parameters a, b and c of function [4] was also high indicating the non-significance of the estimates. Only the parameters of function [6] have the value of ASE below one (0.284 and 0.093), lower than that of any other function.

The Wald Confidence intervals (95%) for these functions were also compared to check the certainty of parameter estimates (Table 5). It is evident that estimates of parameter 'a' of functions [1] and [3] and also of parameter 'a' and 'b' of function [4] were not certain as their confidence intervals were too wide. The Wald confidence intervals for parameter 'a' of function [5] could not

be estimated. The parameter estimates for function [6] were 0.936 and 1.809 and corresponding Wald Confidence intervals were 0.362, 1.510 and 1.620, 1.998, respectively showing the certainty of estimates.

Although the function [1] has the minimum value of MSE and highest value of adj.  $R^2$ , this function failed on the criteria of ASE and Wald confidence intervals. Thus function [6] i.e. allometric function may be selected on the basis of these criteria. The Chi-square test of goodness of fit was also applied for judging adequacy of the model. The fitted model was found to be a good fit as the calculated value of  $\chi^2$  (6.593) was lower than tabulated value (30.14) at 5% level of significance.

The graphical presentation exhibited that observed data are well fitted by this function (Fig.1). The residuals of this fitted function were analyzed graphically for their independence and normality. The autocorrelation plot of residuals depicts their independence (Fig. 2) and the probability plot of residuals clearly shows that residuals were normally distributed (Fig. 3). The scatter plot of Studentized residuals and predicted biomass portrays the homogeneity of variance i.e. variance was constant over the entire range of predicted biomass (Fig. 4).

The fitted allometric model was validated on

**Table 4.** Parameters' estimates and their ASE for functions fitted using D.

Function No.	Parameters' estimate				ASE			
	a	b	c	d	a	b	c	d
[1]	1950.000	3.860	0.088	--	2270.562	0.888	0.021	--
[2]	0.950	-4.221	-0.013	--	8.933	9.019	0.020	--
[3]	1970.000	0.027	2.527	--	2911.262	0.035	1.218	--
[4]	26.175	-25.000	11.990	-0.448	164.047	739.847	122.938	5.963
[5]	4.082	-1.671	-0.224	0.450	x	1.809	0.625	0.512
[6]	0.936	1.809	--	--	0.284	0.093	--	--

x- could not be estimated

**Table 5.** Wald confidence interval for the parameters' estimates for functions fitted using D.

Function No.	Wald Confidence Interval (95%)							
	a (lower)	a (upper)	b (lower)	b (upper)	c (lower)	c (upper)	d (lower)	d (upper)
[1]	-2646.51	6546.51	2.063	5.658	0.045	0.131	--	--
[2]	-17.13	19.03	-22.479	14.037	-0.054	0.028	--	--
[3]	-3918.54	7868.54	-0.045	0.098	0.061	4.992	--	--
[4]	-306.21	358.56	-1524.07	1474.072	-237.10	261.087	-12.531	11.634
[5]	x	x	-5.337	1.995	-1.490	1.043	-0.588	1.488
[6]	0.362	1.510	1.620	1.998	--	--	--	--

x- could not be estimated

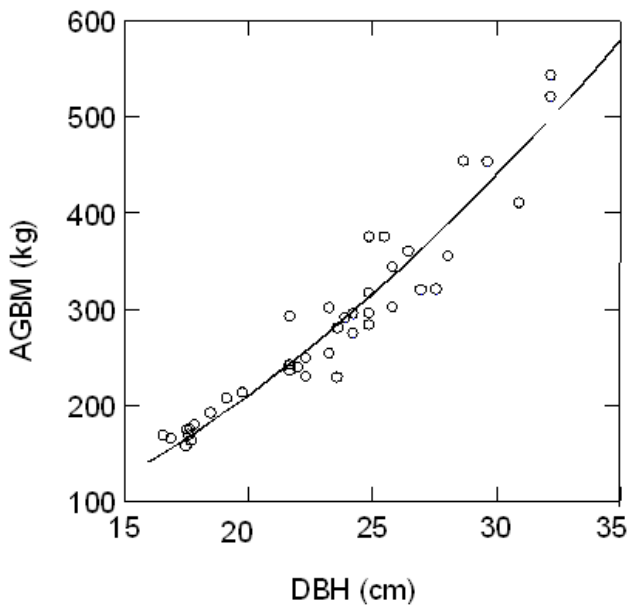


Fig 1. Fitted allometric function using DBH.

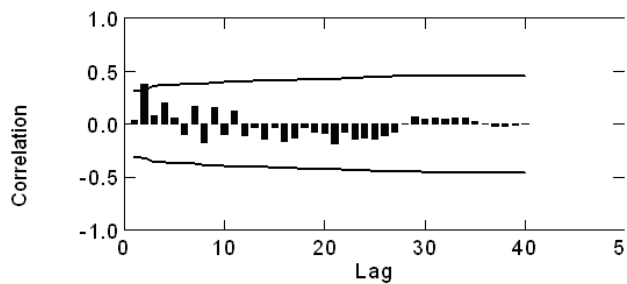


Fig 2. Autocorrelation plot of residuals for allometric function.

data from Poplar trees of age 7 and 8 years for prediction accuracy. The predictions of aboveground biomass were found to be more on negative side with maximum error of -22.83. The minimum error in prediction was only 0.183 by this function. The mean absolute error (MAE) of predictions comes out to be 10.52 indicating that an error of only 10.52 kg in aboveground biomass may be committed by this function. The variance ratio for this fitted function was 0.901 showing lower variability in predicted biomass than actual ones. The model efficiency was found to be 0.949 meaning thereby model is 94.9 percent efficient in predictions.

*Aboveground Biomass – D<sup>2</sup>H Relationship*

Aboveground biomass and D<sup>2</sup>H relationship for Poplar trees was also investigated by fitting non-

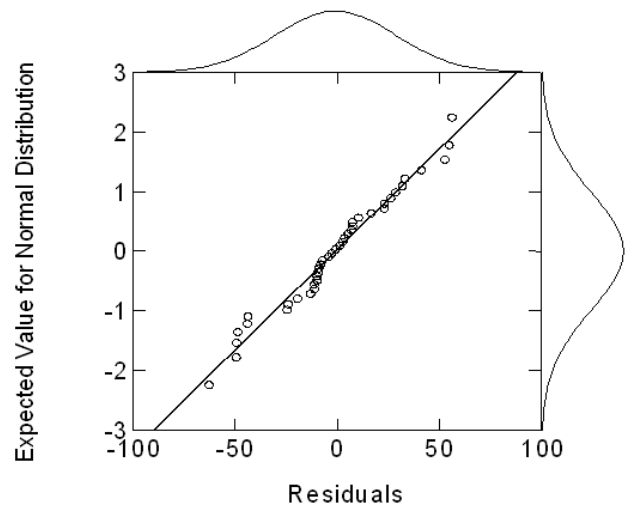


Fig 3. Probability plot of residuals for allometric function.

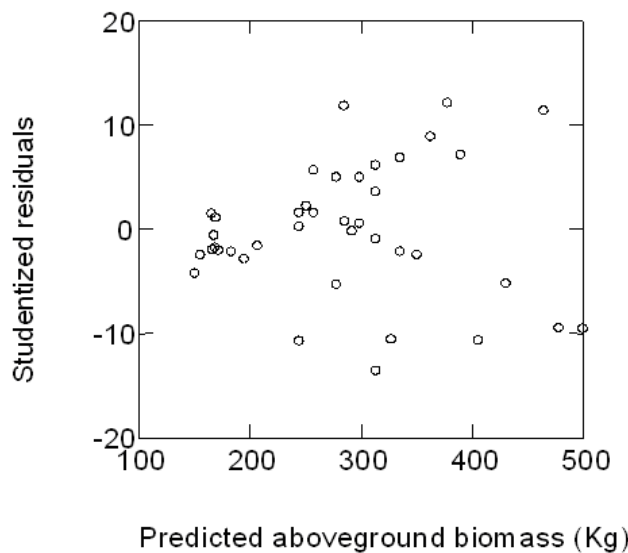


Fig 4. Scatter plot of Studentized residuals for fitted function.

linear functions (Table 1). The mean square error (MSE) and adjusted R<sup>2</sup> for fitted functions are presented in Table 3. It was found that MSE was minimum (359.882) for function [2] and the next minimum value was 360.615 for both functions [4] and [5]. The values of adj. R<sup>2</sup> were very high (> 0.95) for all the functions. This indicated that ‘D<sup>2</sup>H’ would certainly be better explanatory variable than ‘D’. The highest value was 0.965 for functions [4] and [5] followed by function [2] i.e. 0.964.

The parameter estimates and corresponding asymptotic errors (ASE) for fitted functions were also compared (Table 6). The ASE for parameter ‘a’ of functions [2], [3], [4] and [5] was found to be high as also for parameter ‘b’ of functions [4] and [5]. So the estimates of these parameters were non-significant. Only the parameters of functions [1] and [6] have the low value of ASE. The Wald confidence intervals also confirm the uncertainty of parameter ‘a’ of functions [2], [3], [4] and [5] as the intervals were too wide (Table 7). The confidence intervals for parameter ‘b’ of functions [4] and [5] were also wide showing uncertainty of parameter estimate. Thus, functions [1] and [6] i.e. logistic and allometric functions were selected on the basis of these statistical criteria.

For assessing the performance of two models, modified error index (MEI) was used. The two functions were applied on an independent data set and the variability in (D/H) and (H/D) was examined for this data. Since the variability in D/H was higher than that of H/D, so  $D_i/H_i$  was taken as weight ( $W_i$ ) for the  $i^{\text{th}}$  tree. The fitted

logistic and allometric functions with their adjusted  $R^2$ ,  $\chi^2_{\text{cal}}$  and MEI are given in Table 8. It is evident that model 1 has higher value of adj.  $R^2$  and lower values of  $\chi^2_{\text{cal}}$  and MEI than model 2. Hence, the logistic model was finally selected on the basis of these criteria for the prediction of aboveground biomass.

The selected logistic model also exhibits good fit of the observed data graphically (Fig. 5). The residuals of this fitted function were analyzed graphically for their independence and normality. The independence of residuals is depicted by an autocorrelation plot (Fig. 6) and the probability plot of residuals clearly shows that residuals were normally distributed (Fig. 7). The scatter plot of Studentized residuals and predicted biomass portrays the homogeneity of variance i.e. variance was constant over the entire range of predicted biomass (Fig. 8).

The fitted logistic model was validated by predicting the aboveground biomass of trees of age 7 and 8 years. The maximum and minimum error in predicted biomass for these trees comes out to

**Table 6.** Parameters’ estimates and their ASE of functions fitted using  $D^2H$ .

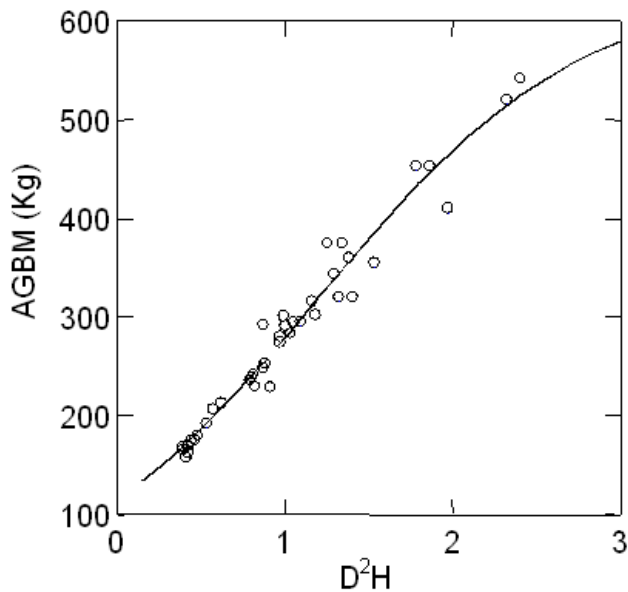
Function No.	Parameters’ estimate				ASE			
	a	b	c	d	a	b	c	d
[1]	639.186	1.515	1.264	--	47.448	0.063	0.121	--
[2]	788.314	2.025	0.675	--	102.62	0.080	0.106	--
[3]	1995.000	0.065	0.700	--	3807.927	0.207	0.092	--
[4]	18.660	-20.261	2.467	-0.156	123.404	354.569	13.676	1.471
[5]	50.723	-20.262	2.467	-0.156	335.448	354.559	13.676	1.471
[6]	287.394	0.675	--	--	3.271	0.023	--	--

**Table 7.** Wald confidence interval for the parameters’ estimates for functions fitted using  $D^2H$ .

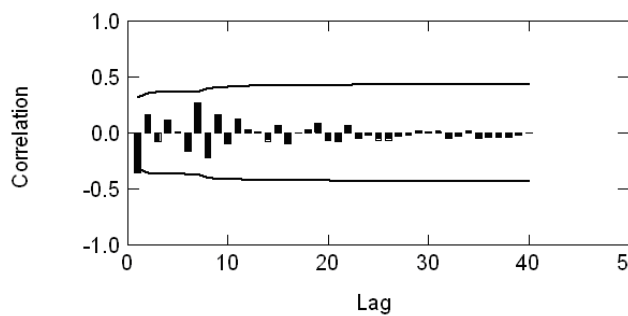
Function No.	Wald Confidence Interval (95%)							
	a (lower)	a (upper)	b (lower)	b (upper)	c (lower)	c (upper)	d (lower)	d (upper)
[1]	543.048	735.325	1.388	1.642	1.019	1.508	--	--
[2]	580.387	996.241	1.862	2.187	0.460	0.890	--	--
[3]	-5703.74	9703.74	-0.355	0.485	0.514	0.885	--	--
[4]	-231.61	268.936	-739.34	698.816	-25.268	30.202	-3.139	2.827
[5]	-629.59	731.043	-739.34	698.816	-25.268	30.202	-3.139	2.827
[6]	280.772	294.016	0.629	0.721	--	--	--	--

**Table 8.** Comparison of two selected models for aboveground biomass.

S.N.	Fitted models	Adj. $R^2$	$\chi^2$	MEI
1.	$B = 639.186 (1 + \exp(1.515 - 1.261 D^2H))^{-1}$	0.963	3.069	7.175
2.	$B = 287.394 * (D^2H)^{0.675}$	0.961	3.472	7.511



**Fig 5.** Fitted logistic function for aboveground biomass.

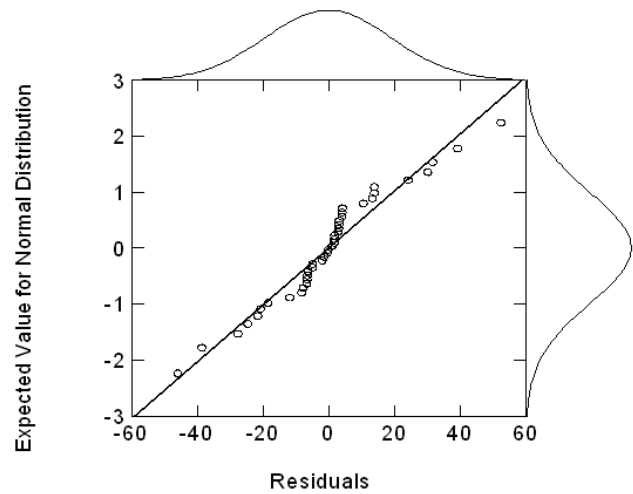


**Fig 6.** Autocorrelation plot of residuals for logistic function.

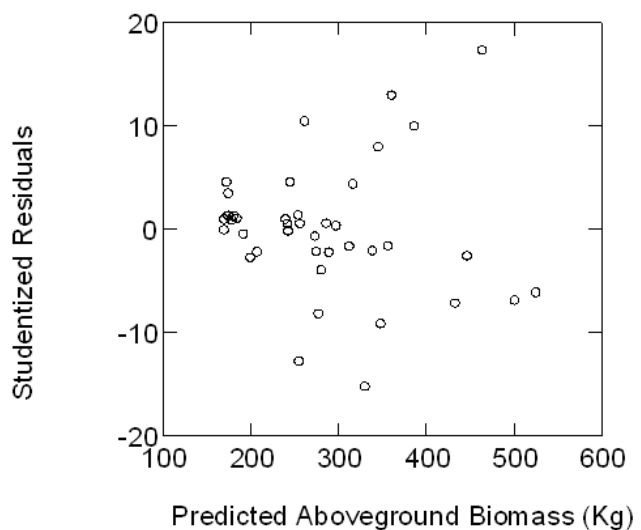
be 14.31 and -0.36, respectively. The mean absolute error (MAE) comes out to be 7.20 indicating that an error of only 7.20 kg may be committed in predicted aboveground biomass. The variance ratio (VR) was almost one (0.98) showing same variability in predicted and actual biomass of trees. The model efficiency comes out to be 98.4 percent.

## Conclusions

Aboveground biomass estimation of *Populus deltoides* without harvesting the tree is of much importance for foresters and wood merchants. The models developed earlier were either judged on single criterion of  $R^2$  only or not statistically validated on an independent data set. The



**Fig 7.** Normal probability of residuals for logistic function.



**Fig 8.** Plot of Studentized residuals for logistic function.

proposed two models were judged on statistical criteria such as adjusted  $R^2$ , mean square error, asymptotic standard error of parameter estimates and Wald confidence interval. Moreover, the proposed models were also validated on an independent data set for examining their accuracy of prediction. Hence these models may be used for estimating/predicting the aboveground biomass of *Populus deltoides* tree in Haryana region.

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